Engineering Mechanics

Chapter: 1 Force Concept

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In This Chapter We Cover the Following Topics

1.1 What is force
1.2 Basic definitions
1.3 Normal reaction
1.4 Parallelogram Law
1.5 Lame's Theorem
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1.7 Equilibrium of Particle and Body
1.8 Force System
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1.10 Varignon's Theorem
1.11 Newton's Law

Please welcome for any correction or misprint in the entire manuscript and your valuable suggestions kindly mail us *brijrbedu@gmail.com*.

1.1 WHAT IS FORCE?

Force is a physical quantity by the action of which a body may change its state of motion.
In engineering mechanics force is the action of one body on another. A force tends to move a body in the direction of its action,
A force is characterized by its point of application, magnitude, and direction, i.e. a force is a vector quantity.
According to Newton's second law force is defined as the rate of change of linear momentum w.r.t. time.

$$F = m.a$$

1.2 BASIC DEFINITIONS

Space
Space is the geometrical region occupied by bodies whose positions are described by linear and angular measurement relative to coordinate systems. For three dimensional problems there are three independent coordinates are needed. For two dimensional problems only two coordinates are required.

Particle
A particle may be defined as a body (object) has mass but no size (neglected), such body cannot exists theoretically, but when dealing with problems involving distance considerably larger when compared to the size of the body. For example a bomber aeroplane is a particle for a gunner operating from ground.
In the mathematical sense, a particle is a body whose dimensions are considered to be near zero so that it analyze as a mass concentrated at a point. A body may tread as a particle when its dimensions are irrelevant to describe its position or the action of forces applied to it. For example the size of earth is insignificant compared to the size of its orbits and therefore the earth can be modeled as a particle when studying its orbital motion. When a body is idealized as a particle, the principles of mechanics reduce to rather simplified form since the geometry of the body will not be involved in the analysis of the problem.

Rigid Body
A rigid body may be defined a body in which the relative positions of any two particles do not change under the action of forces means the distance between two points/particles remain same before and after applying external forces.
As a result the material properties of any body that is assumed to be rigid will not have to be considered while analyzing the forces acting on the body. In most cases the actual deformations occurring in the structures, machines, mechanisms etc are relatively small and therefore the rigid body assumption is suitable for analysis

Principle of Transmissibility of Forces
Its state that if a force acting at a point on a rigid body is shifted to any other point which is on the line of action of the force, the external effect of the force on the body remains unchanged.

The state of rest of motion of a rigid body is unaltered if a force acting in the body is replaced by another force of the same magnitude and direction but acting anywhere on the body along the line of action of the replaced force.
For ***example*** the force F acting on a rigid body at point A. According to the principle of transmissibility of forces, this force has the same effect on the body as a force F applied at point B [Diagram 1.1].

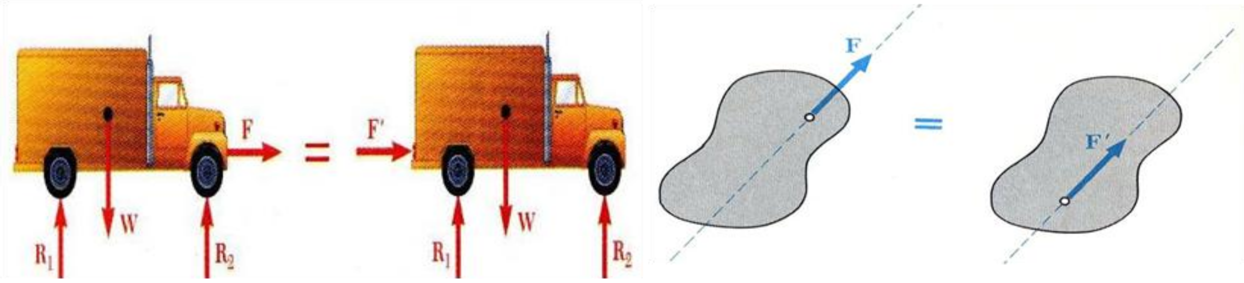


Diagram 1.1

The following two points should be considered while using this principle.

1. In engineering mechanics we deal with only rigid bodies. If deformation of the body is to be considered in a problem. The law of transmissibility of forces will not hold good.
2. By transmission of the force only the state of the body is unaltered, but not the internal stresses which may develop in the body

Therefore this law can be applied only to problems in which rigid bodies are Involved

Free Body Diagram

A diagram or sketch of the body in which the body under consideration is freed from the contact surface (surrounding) and all the forces acting on it (including reactions at contact surface) are drawn is called free body diagram. Free body diagram for few cases are shown in below

1.3 NORMAL REACTION

Let us consider a body A of weight "W" rest over another surface B and a force F acting on the body to slide the body on the surface B as shown in Diagram 1.2. A little concentration will show that the body A presses the surface B downward equal to weight of the body and in reaction surface B lift the body in upward direction of the same magnitude but in opposite direction therefore the body in equilibrium this upward reaction is termed as normal reaction and it is denoted by R or N.

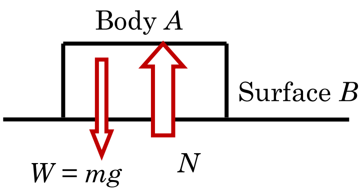


Diagram 1.2

Note: It is noted the weight W is not always perpendicular to the surface of contact and hence normal reaction N is not equal to the weight W of body in such a case the normal reaction is equal to the component of weight perpendicular to surface.

1.4 PARALLELOGRAM LAW

According to parallelogram method 'If two forces (vectors) are acting simultaneously on a particle be represented (in magnitude and direction) by two adjacent sides of a parallelogram, their resultant may represent (in magnitude and direction) by the diagonal of the parallelogram passing through the point.
In other words when two forces are acting at a point such that they can by represented by the adjacent sides of a parallelogram then their resultant will be equal to that diagonal of the parallelogram which passed through the same point.

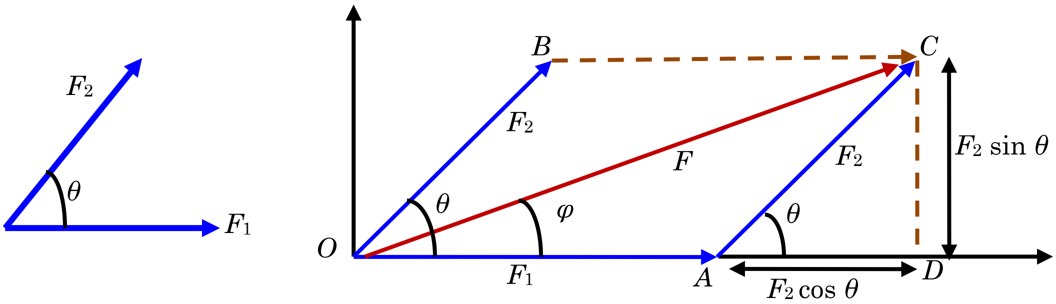


Diagram 1.3

In the parallelogram $OABC$, from point C drop a perpendicular CD to meet OA at D as shown in Diagram 1.3.
In parallelogram $OABC$, $OA = F_1$; $OB = F_2$; Angle $AOB = \theta$
Now consider the ΔCAD in which Angle $CAD = \theta$; $AC = F_2$
By resolving the vector F_2 we have,

$$CD = F_2 \sin\theta \text{ and } AD = F_2 \cos\theta$$

Now consider ΔOCD Angle $DOC = \varphi$. Angle $ODC = 90^\circ$
According to Pythagoras theorem

$$(\text{Hyp})^2 = (\text{per})^2 + (\text{base})^2$$

$$OC^2 = DC^2 + OD^2. \quad \text{or} \quad OC^2 = DC^2 + (OA + AD)^2$$

$$F^2 = F_2^2 \sin^2\theta + (F_1 + F_2\cos\theta)^2$$

$$F^2 = F_2^2 \sin^2\theta + F_1^2 + F_2^2\cos^2\theta + 2F_1F_2\cos\theta$$

$$F^2 = F_2^2(\sin^2\theta + \cos^2\theta) + F_1^2 + 2F_1F_2\cos\theta$$

$$F^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$$

Triangle Method/ Triangle Law of Forces

According to triangle law or method" If two forces acting simultaneously on a particle by represented (in magnitude and direction) by the two sides of a triangle taken in order their resultant is represented (in magnitude and direction) by the third side of triangle taken in opposite order.
If two forces are acting on a body such that they can be represented by the two adjacent sides of a triangle taken in the same order, then their resultant will be equal to the third side (enclosing side) of that triangle taken in the opposite order.

1.5 LAME'S THEOREM

It states, "If there are three forces acting at a point be in equilibrium then each force is proportional to the sine of the angle between the other two forces".

Let three force F_1, F_2 and F_3 acting at a point and the opposite angles to three forces are γ , β, and α as shown in Diagram 1.4.

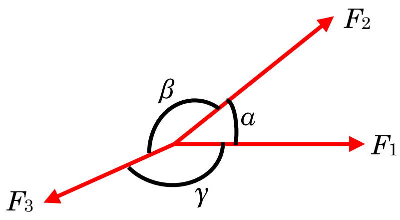


Diagram 1.4

According to Lami's theorem

$$\frac{F_1}{\sin\beta} = \text{Const.}; \ \frac{F_2}{\sin\gamma} = \text{Const.}; \ \frac{F_3}{\sin\alpha} = \text{Const.}$$

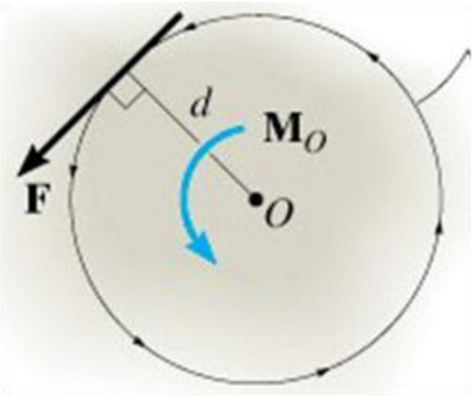
$$\frac{F_1}{\sin\beta} = \frac{F_2}{\sin\gamma} = \frac{F_3}{\sin\alpha}$$

1.6 MOMENT OF A FORCE

The tendency of a force to move the body in the direction of its application a force can tend to rotate a body about an axis. This axis may be any line which is neither intersects nor parallel to the line of the action of the force. This rational tendency of force is known as the moment of force.

As a familiar example of the concept of moment, consider the pipe wrench as shown in Diagram 1.5. One effect of the force applied perpendicular to the handle of the wrench is the tendency to rotate the pipe about its vertical axis. The magnitude of this tendency depends on both the magnitude of the force and the effective length d of the wrench handle.

Common experience shown that a pull which is not perpendicular to the wrench handle is less effective than the right angle pull. Mathematically this tendency of force (moment) is calculated by multiplying force to the moment arm (d)



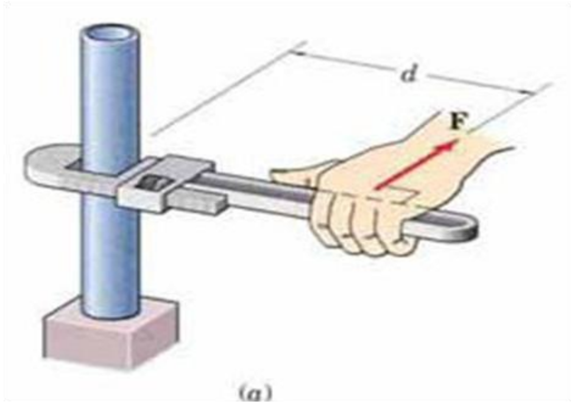


Diagram 1.5

Moment about a Point

Consider following body (two dimensional) acted by a force F in its plane. The magnitude of moment or tendency of the force to rotate the body about the axis O-O perpendicular to the plane of the body is proportional both to the magnitude of the force and to the moment arm d, therefore magnitude of the moment is defined as the product of force and moment arm.

$$\text{Moment} = \text{moment arm} \times \text{Force}$$
$$M = d \times F$$

where d = moment arm and F = magnitude of force

Moment arm is defined as the perpendicular distance between axis of rotation and the line of action of force.

Couple

When two parallel forces that have the same magnitude but opposite direction is known as couple. The couple is separated by perpendicular distance. As matter of fact a couple is unable to produce any straight-line motion but it produces rotation in the body on which it acts. So couple can be defined as unlike parallel forces of same magnitude but opposite direction which produce rotation about a specific direction and whose resultant is zero

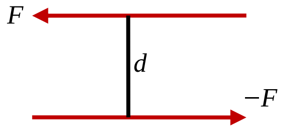


Diagram 1.6

Application of Couple

1. To open or close the valves or bottle head, tap etc
2. To wind up a clock.
3. To Move the paddles of a bicycle
4. Turning a key in lock for open and closing.

Couple Arm: The perpendicular distance between the lines of action of the two and opposite parallel forces is known as arm of the couple.

Moment of couple or couple moment: The moment of the couple is the product of the force (one of the force of the two equal and opposite parallel forces) and the arm of the couple. Mathematically

$$\text{Moment of couple} = \text{moment arm x Force}$$
$$\text{Moment of couple} = r \times F$$

1.7 EQUILIBRIUM OF PARTICLE AND BODY

Equilibrium of a Particle

When the resultant of all forces acting on a particle is zero, the particle is said to be in equilibrium. A particle which is acted an upon two forces

Newton's First Law

If the resultant force on a particle is zero, the particle will remain at rest or will continue at constant speed in a straight line.

Equilibrium

A particle is in equilibrium if it is at rest if originally at rest or has a constant velocity if originally in motion. The term equilibrium or static equilibrium is used to describe an object at rest. To maintain equilibrium it is necessary to satisfy Newton's first law of motion, which requires the resultant force acting on particle to be equal to zero. That is

$$\Sigma F = 0 \qquad (a)$$

where ΣF = Sum of all the forces acting on the particle which is necessary condition for equilibrium. This follows from Newton's second law of motion, which can be written as

$$\Sigma F = ma.$$

Put in equation (a) $ma = 0$

Therefore the particle acceleration $a = 0$. Consequently the particle indeed moves with constant velocity or at rest.

1.8 FORCE SYSTEM

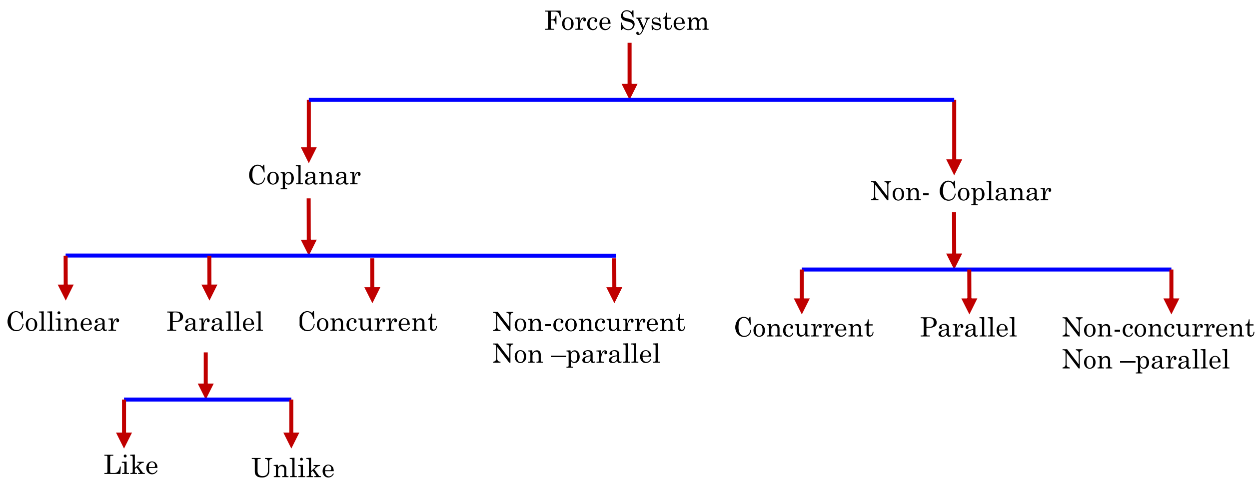
Force exerted on body has following two effects

1. The ***external effect***, which is tendency to change the motion of the body or to develop resisting forces in the body
2. The ***internal effect***, which is the tendency to deform the body.

If the force system acting on a body produces no external effect, the forces are said to be in ***balance*** and the body experience no change in motion is said to be in ***equilibrium***.

Systems of Forces

When numbers of forces acting on the body then it is said to be system of forces



- *The word collinear stands for the forces which are having common lines of action .*
- *The word concurrent stands for the forces which intersect at a common point.*

Coplanar Collinear

As shown in Diagram 1.7. Three forces F_1, F_2, F_3 acting on a plane, these three forces are in the same line i.e, these three forces are in the same a common line of action. The system of forces is known as coplanar collinear force system. This means that in coplanar collinear system of forces, all the forces act in the same plane & have a common line.

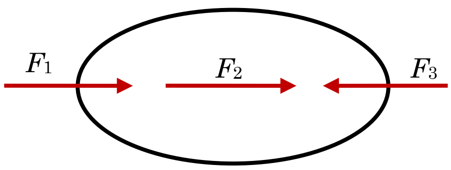


Diagram 1.7

Coplanar Concurrent

As shown in the Diagram 1.8. Three forces F_1, F_2, F_3 acting in a plane & these forces intersect or meet at a common Point O. This system of forces is known as coplanar Concurrent force system. Hence, in coplanar concurrent System of forces, all the forces acting in the same plane & they intersect at a common point.

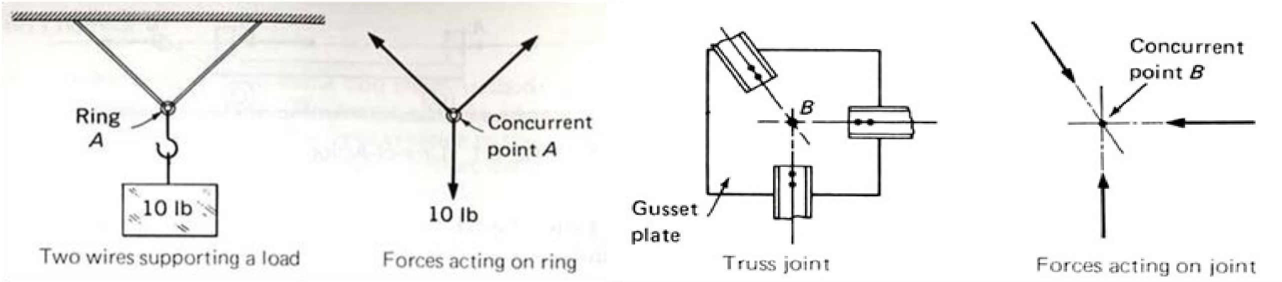


Diagram 1.8

Coplanar Parallel

As shown in a Diagram 1.9 three forces F_1, F_2, F_3 acting in a Plane & these forces are parallel. This system of forces is known as coplanar parallel force system. Hence, in coplanar Parallel force system, all the forces are act in the same plane are parallel.

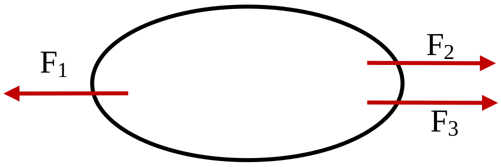


Diagram 1.9

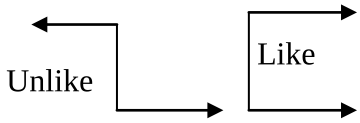


Diagram 1.10

Like* II *force: The II forces which are acting in the same direction are known as like II forces [Diagram 1.10].
Unlike forces: The II forces which are acting in the opposite direction are known as unlike forces [Diagram 1.10].

Coplanar Non –Concurrent Non Parallel

As shown in Diagram 1.11 four forces F_1, F_2, F_3 acting in a plane. These lines of these forces lie in the same plane but they are neither parallel nor meet or intersect at a common point. This system of forces is known as coplanar non concurrent non parallel force system. hence in coplanar non concurrent non parallel system of forces, all the forces act in the same plane but the forces are neither parallel nor meet at a common point, this system is also known as general system of forces.

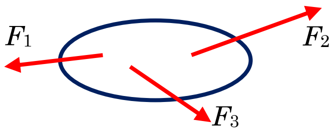


Diagram 1.11

1.9 PRINCIPLE OF MOMENTS

It states that the moment of the resultant of a number, of forces about any point is equal to the algebraic sum of the moments of all the forces of the system about the same point.

Proof: As shown in Diagram 1.12 two forces acting at a point *O*. These forces are represented in magnitude and direction by *OA* and *OB*.

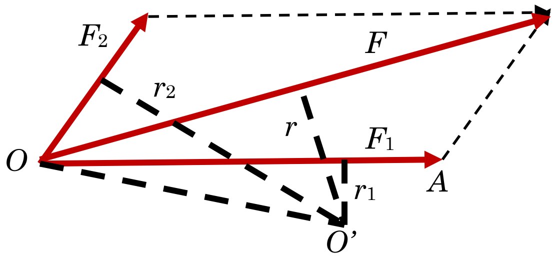


Diagram 1.12

Their resultant *F* is represented in magnitude & direction by *OC* which is diagonal of parallelogram *OACB*.
Let *O'* is the point in the plane about which moments of F_1, F_2 and *F* are to be determined.
From point *O'* draw perpendicular on *OA, OC* and *OB*.
According to principle

$$Fr = F_1 r_1 + F_2 r_2$$

1.10 VARIGNON'S THEOREM

It's states that the moment of a force about any point is equal to the algebraic sum of the moments of its components about that point.

Proof: Consider a force *F* acting at a point *A* & having component F_1 & F_2 in any direction. Now consider any point *O* lying in the plane of the forces, as the moment centre [Diagram 1.13].

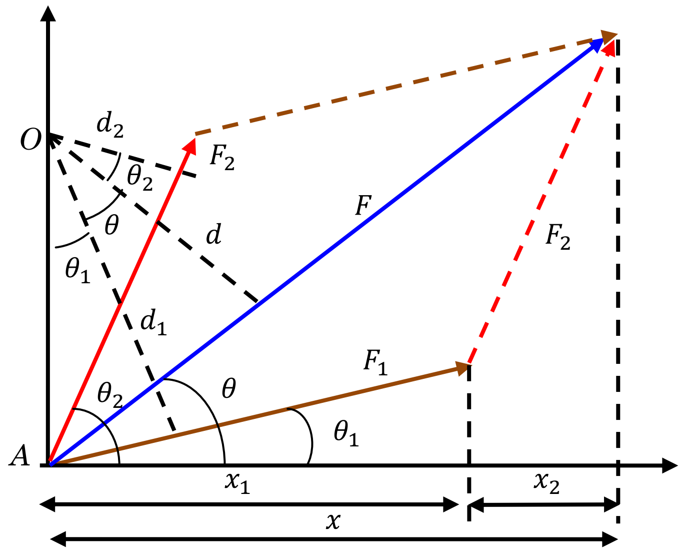


Diagram 1.13

Now, the moment of force *F* about *O* will be

$$F.d = F\,(OA \cos\theta) = OA.F\cos\theta = OA.\,x \qquad [1.1]$$

Now moment of force F_1 about *O*

$$F_1 d_1 = F_1\,(OA \cos\theta_1) = OA\,F_1 \cos\theta_1 = OA\,x_1 \qquad [1.2]$$

Moment of force F_2 about *O*

$$F_2d_2 = F_2\,(OA\cos\theta_2) = OA\,F_2\cos\theta_2 = OA\,x_2 \quad [1.3]$$

Adding Eqns. (1.2), and (1.3)

$$F_1d_1 + F_2d_2 = OA\,(x_1 + x_2)$$

$$Fd = OA\,(x_1 + x_2)$$

From Eqn. (1.1)

$$OA\,x = OA\,(x_1 + x_2)$$

$$x = x_1 + x_2$$

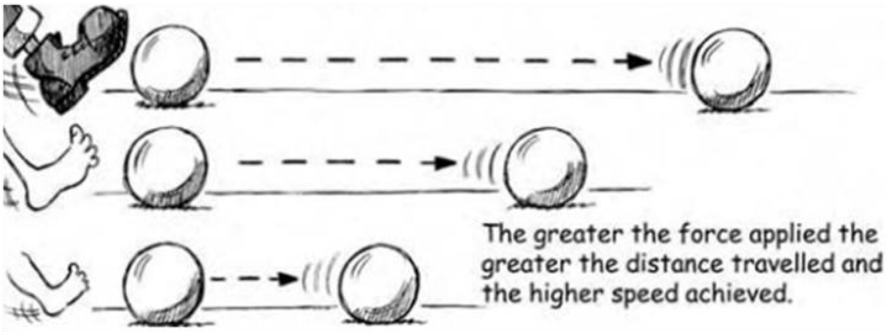
$$Fd = F_1d_1 + F_2d_2$$

Note- The principle of moments (or varigon's principle) is not restricted to only two concurrent forces but is also applicable to any coplanar force system i.e. concurrent or non-concurrent or parallel force system.

1.11 NEWTON'S LAW

***Newton's First Law*:** Every particle continues in a state of rest or uniform motion in a straight line unless it is compelled to change that state by forces imposed on it

***Newton's Second Law*:** The change of motion is proportional to the natural force impressed and is made in a direction of the straight line in which the force is impressed



***Newton's Third Law*:** The forces of action and reaction between two particles have the same magnitude and line of action with opposite sense.