

# UPSC Civil Services Main 1999 - Mathematics

## Algebra

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**Question 1(a)** *If  $\phi$  is a homomorphism of  $G$  into  $H$  with kernel  $K$ , then show that  $K$  is a normal subgroup of  $G$ .*

**Solution.** See Lemma 2.7.3 page 57 of Algebra by Herstein. ■

**Question 1(b)** *If  $p$  is a prime number  $p^\alpha \mid o(G)$ , then prove that  $G$  has a subgroup of order  $p^\alpha$ .*

**Solution.** This is Sylow's theorem. See theorem 2.12.1 page 92 of Algebra by Herstein. ■

**Question 1(c)** *Let  $R$  be a commutative ring with unit element whose only ideals are the zero ideal and  $R$  itself. Show that  $R$  is a field.*

**Solution.** See Lemma 3.5.1 Page 137 of Algebra by Herstein. ■