Problems of Practices
Of
Mechanics of Solids
10- Energy Methods

Prepared By

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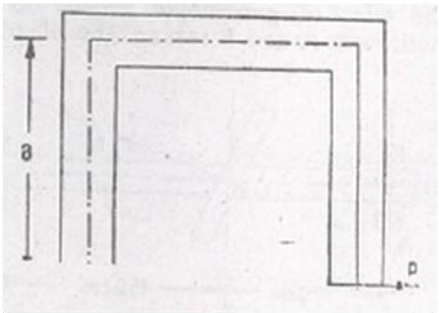
1. State the theorem of Castigliano.

2. Using the above theorem find the horizontal displacement along the load line of the frame shown in Fig. 3, considering the deflection due to bending only. The moment of inertia is the same for all sections.



3. A 1000 mm long bar is subjected to an axial pull P which induces a maximum stress of 1500 kg/cm^2. The area of cross-section of the bar is 2 cm^2 over a length of 950 mm and for the central 50 mm length the sectional area is equal to 1.0 cm^2. Assuming that E for the bar material is 20×10^5 kg/cm^2, calculate the strain energy stored in the bar.

4. Using the Castigliano theorem, calculate the vertical deflection δ at the middle of a simply supported beam which carries a uniformly distributed load of intensity w over the full span. The flexural rigidity EI of the beam is constant and only strain energy of bending is to considered.

5. A uniform cantilever beam with flexural rigidity EI is carrying a uniformly distributed load of magnitude w per unit length on its half length from mid span to free end. The length of cantilever is l. Derive an expression for bending deflection at the free end. You may use First Theorem of Castigliano.

6. A shaft is supported by two anti-friction bearings with loads of 140 Newton's each acting at points B and F as shown in Fig.3. The portion of shaft between B and C has diameter of $2D$ compared to a diameter D for the portion of the shaft between A and B; and between C and F. Using the Castigliano's theorem, determine the deflection of shaft at point B and F.

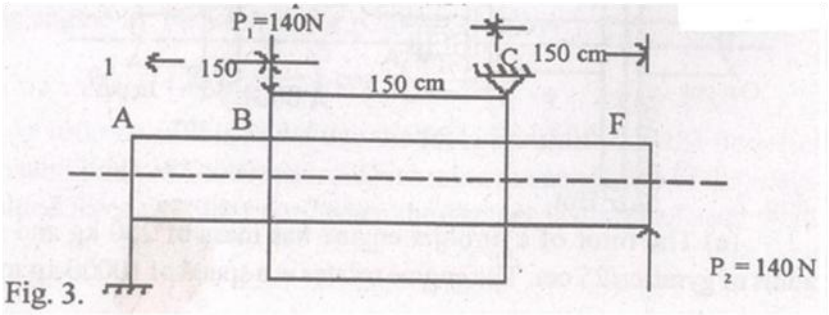


Fig. 3.

7. A steel tube having outside and inside diameters of 10 cm and 6 cm respectively is bent into the form of a quadrant of 2 m radius as shown in Fig. 4. One end is rigidly attached to a horizontal base plate to which a tangent to that end is perpendicular, and the free end supports a load of 1000 N. Determine the vertical and horizontal deflections of the free end under this load using Castiglione's theorem. Given $E = 2 \times 10^{11}$ N/m^2.

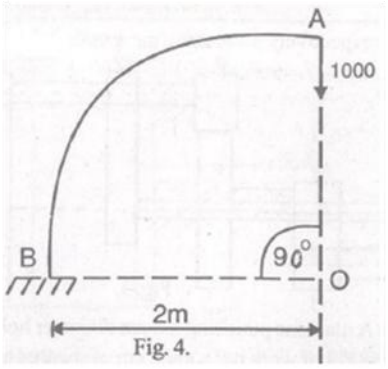


Fig. 4.

8. A shaft circular in section and of length l is subjected to a variable torque given by $(kx^2)/l$ where x is the distance measured from one end of the shaft and k is a constant. Find the angle of twist for the shaft by using Castiglione's theorem. Torsional rigidity of the shaft G see in Fig. 5.

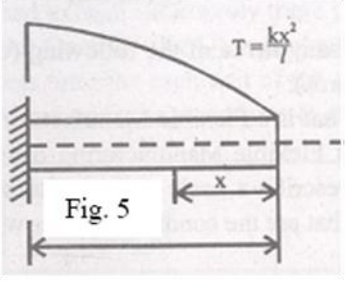


Fig. 5

9. A circular bar of diameter d is bent at right angle. It is fixed at one end and a load W is applied at the other end as shown in the figure 1. Determine the deflection under the load W if E & G are Young's and Shear Moduli of the materials.

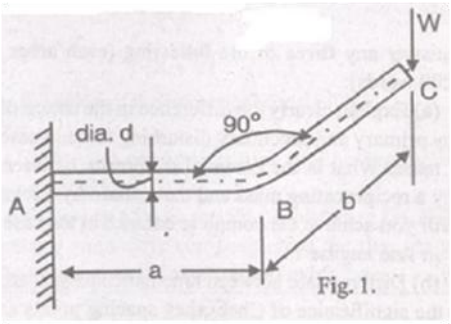


Fig. 1.

10. A steel bar of diameter 60 mm and length 300 mm is subjected to an axial compressive load of 50 kN. To what diameter the middle one-third length of the bar be reduced in order to increase the stored energy by 50%?

11. An ISJB 150 rolled steel joist is simply supported over a span of 4 m. A weight of 380 N is dropped onto the middle of the beam, producing an instantaneous stress of 80 N/mm^2. Calculate the height from which the weight was dropped and the maximum deflection in the beam.
Assume I = 322.1 cm^4 and E = 200 GPa, Depth of the beam = 150 mm.

12. During a routine manufacturing operation, a rod AB of 20 mm diameter and 1.5 m long, must acquire an elastic strain-energy of 13.6 N-m. Using E = 200 GPa, determine the yield stress of steel, if the factor of safety with respect to the permanent deformation is to be five.

13. A simply supported beam of span L and rectangular cross-section ($b \times h$) is loaded by uniformly distributed load of w N/unit length. If σ_{max} is the maximum bending stress and A is the cross-sectional area, then show that the elastic strain energy due to bending of the beam will be given by

$$\left(\frac{\sigma_{max}^2}{2E}\right)\left(\frac{8}{45} \times \text{Volume of the beam}\right)$$

Sketch the beam.

14. A semicircular steel ring of mean radius 300 mm is suspended vertically with the top end fixed as shown in the above figure and carries a vertical load of 200 N at the lowest point. Calculate the vertical deflection of the lower end if the ring is of rectangular cross-section 20 mm thick and 30 mm wide. Value of Elastic modulus is 2 × 10^5 N/mm^2. Influence of circumferential and shearing forces may be neglected.