Problems of Practices
Of
Mechanics of Solids
2- Stresses and Strain on Oblique Plane

Prepared By

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1. At a given point in a machine element, the following stresses were evaluated: 100 MPa T and zero shear on a horizontal plane and 50 MPa C on a plane perpendicular to this plane. (Fig. 4). Determine the stresses at this point on a plane having a slope of 3 vertical to 4 horizontal.

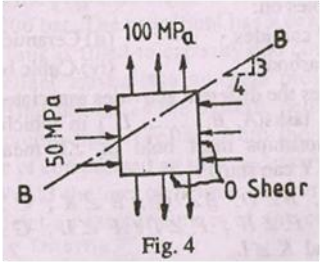


Fig. 4

2. At a point in an elastic material the stresses on three mutually perpendicular planes are as follows:
First plane: 50 MN/m² tensile and 40 MN/m² shear.
Second plane: 30 MN/m² compressive and 40 MN/m² complimentary shear.
Third plane: No stress.
Find
(i) the position of principal planes and the magnitude of principal stresses,

(ii) the position of planes on which maximum shear stress act calculate the normal and shear stress on them.

3. Do you think that a plane stress system gives rise to a plane strain system always? Give reasons for your answer. Three principal stresses σ_{11}, σ_{22}, σ_{33} are acting on the three faces of a cube respectively. Express the resulting three principal strains in terms of the principal stresses.

4. Draw Mohr's circle for a 2-D case stress field subjected to

(i) Pure shear (ii) Pure bi-axial tension

(iii) Pure uni-axial compression (iv) Pure uni-axial tension.

5. The normal stresses at a point in a strained material across two planes at right angles to each other are 120 N/mm^2 tensile and 60 N/mm^2 compressive. The shear stress on these planes is 40 N/mm^2. Find:

(i) Principal stresses and principal planes; and

(ii) The direct and shear stresses on a plane inclined at 30° to the vertical.

6. A thin cylinder with closed ends has an internal diameter of 50 mm and a wall thickness of 2.5 mm. It is subjected to an axial pull of 10 kN and a torque of 500 Nm while under an internal pressure of 6 MN/m^2.

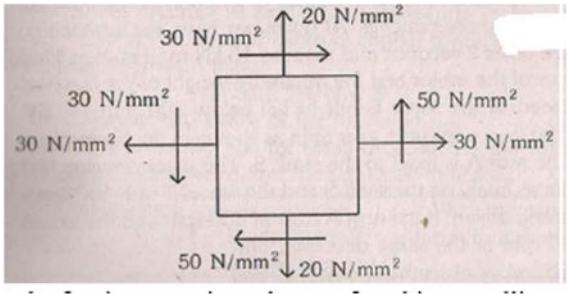
(i) Determine the principal stresses in the tube and the maximum shear stress.

(ii) Represent the stress configuration on a square element taken in the load direction with direction and magnitude indicated (schematic).

(iii) Sketch the Mohr's stress circle.

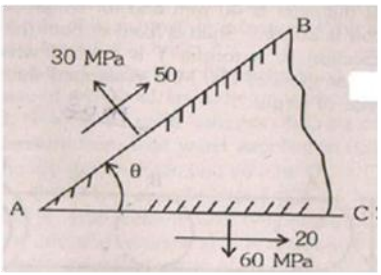
7. Direct tensile stresses of 120 MN/m^2 and 70 MN/m^2 act on a body on mutually perpendicular planes. What is the magnitude of shearing stress that can be applied so that the major principal stress at the point does not exceed 135 MN/m^2? Determine the value of minor principal stress and the maximum shear stress.

8. The figure given below shows the state of stress at a point namely 30 N/mm^2 tensile in *X*-direction. 20 N/mm^2 tensile in *Y*-direction and a shear stress of 50 N/mm^2. Find the location of the principal planes, principal stresses and the maximum shear stress graphically or otherwise.



9. Two Planes *AB* and *BC* which are at right angles are acted upon by tensile stress of 140 N/mm^2 and a compressive stress of 70 N/mm^2 respectively and also by shear stress 35 N/mm^2. Determine the principal stresses and principal planes. Find also the maximum shear stress and planes on which they act.
Sketch the Mohr circle and mark the relevant data.

10. At a point *A* in a strained material, planes *AB* and *AC* pass through the point as shown in the above figure. Normal and shear stresses on plane *AB* are 30 MPa and 50 MPa respectively as shown. Normal and shear stresses on plane *AC* are 60 MPa and 20 MPa respectively as shown. By graphical or analytical method, determine angle between the planes *AB* and *AC*.



11. A tension member with a cross-sectional area of 30 mm² resists a load of 80 kN. Calculate the normal and shear stresses on the plane of maximum shear stress.

12. For the following states of stress, show the stresses on two given planes at right angles of an element. Find the magnitude and directions of the principal stresses and the maximum shear stresses in each case.

(i) Simple uniaxial tension.

(ii) Pure equal normal stresses on given planes.

(iii) Pure shear stresses on given planes.

13. The magnitudes of normal stresses on two mutually perpendicular planes, at a point in an elastic body are 60 MPa compressive and 80 MPa tensile respectively. Find the magnitudes of shearing stresses on these planes if the magnitude of one of the principal stresses is 100 MPa tensile. Find also the magnitude of the other principal stress at this point.

14. At a point in a material under stress, the intensity of the resultant stress on a certain plane is 50 MN/m² (tensile) inclined at 30° to the normal of that plane. (Refer fig. 2) The stress on a plane at right angles to this plane has a normal tensile component of intensity of 30 MN/m². Find

(i) the resultant stress on the second plane,

(ii) The principal planes and stresses,

(iii) Maximum shear stress and the plane on which it occurs.

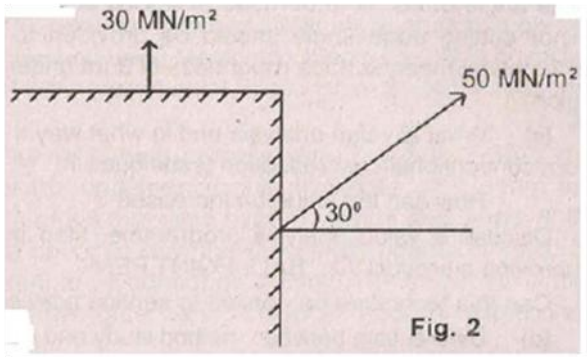
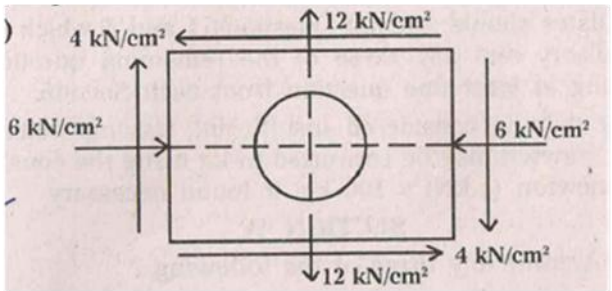


Fig. 2

15. A circle of 100 mm diameter is inscribed on a steel plate before it is stressed. Then the plate is loaded so as to produce stresses as shown in the figure and the circle is deformed into an ellipse. Determine the lengths of major and minor axes of the ellipse and their directions. Take modulus of elasticity of steel as 2.1 MN/cm² and Poisson's ratio 0.28.



16. A rectangular plate of thickness 10 mm carries tensile normal stresses of σ_1 = 600 MPa and σ_2 = 200 MPa on two perpendicular planes on which there are no shear stresses. Obtain the change in thickness of the plate. Take $E = 200 \times 10^9$ Pa and $G = 80 \times 10^9$ Pa.

17. At a point in a strained material, the resultant stress on a vertical plane is 100 MPa (tensile) making an angle of 30° (clockwise) with the normal to the plane. On the horizontal plane through the point, the resultant stress is compressive and makes 60° angle (clockwise) with the normal.

(i) Determine normal and shear stresses acting on the perpendicular planes.

(ii) Obtain principal stresses and principal planes.

(iii) Calculate maximum shear stress and normal stress on the planes of maximum shear stress.

(iv) On a properly oriented element, show principal planes and planes of maximum shear stress.

18. Under which condition of the state of stress at a point in the two dimensions, the Mohr's circle will be reduced to a point?

19. At a point in a loaded structure, a pure shear stress state of $\tau = \pm$ 400 MPa prevails on two given planes at right angles, (i) What would be the state of stress across the planes of an element taken at ±45° to the given planes? (ii) What are the magnitudes of these stresses?

20. For what condition of stresses Mohr's circle is having zero radius.

21. Stresses on two perpendicular planes, at a point, are given in Fig. 7. What are the directions of principal planes with respect to plane *BC*? What are the principal strains on principal planes, if E = 67 kN/mm², ν = 0.33?

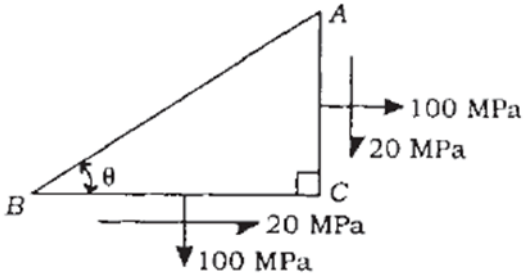


Fig. 7

22. The Mohr's circle for a plane stress is a circle of radius R with its origin at +2R on σ axis. Sketch the Mohr's circle and determine σ_{max}, σ_{min}, σ_{av}, $(\tau_{xy})_{max}$ for this situation.

23. The state of stress at a point in a loaded machine member is given by the principal stresses.

$$\sigma_1 = 600 \text{ MPa}, \sigma_2 = 0 \text{ and } \sigma_3 = -600 \text{ MPa}.$$

(i) What is the magnitude of the maximum shear stress?

(ii) What is the inclination of the plane on which the maximum shear stress acts with respect to the plane on which the maximum principal stress σ_1 acts?

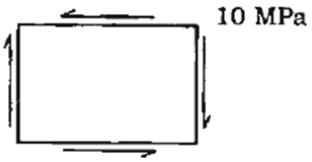
24. The state of stress at a given point in a loaded component is given by σ_x, σ_y and τ_{xy} and the principal stresses at the point are σ_1 and σ_2. If the new axes at the point are changed to x' and y' such that state of stress w.r.t. the new axes are $\sigma_{x'}$, $\sigma_{y'}$ and $\tau_{x'y'}$.

(i) Show that $\sigma_x + \sigma_y = \sigma_{x'} + \sigma_{y'} - \sigma_1 + \sigma_2$.

(ii) Sketch the Mohr's stress circle for pure shear stress state and hydrostatic state of stress.

(iii) A cast iron bar of 30 mm diameter and 1 m length is subjected to equal and opposite torque T applied at its ends. If tensile strength of cast iron is 80 MPa, what is the maximum value of torque T? Show the plane of failure for the applied sense of torque T.

25. An infinitesimal element of a body subjected to plane stress is found to have the state of stress as shown below:



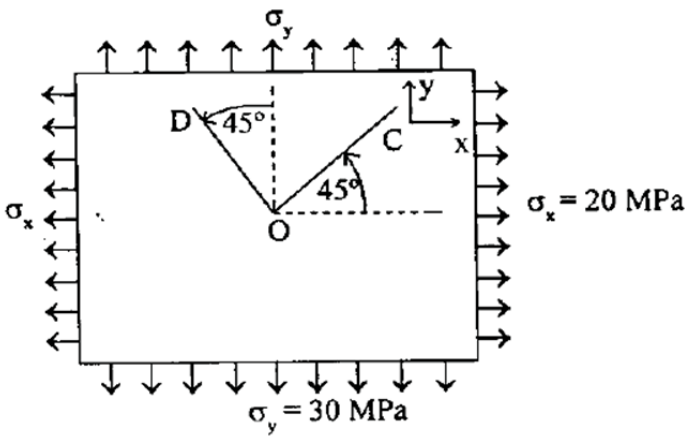
Draw Mohr's circle for the above state of stress. What is this state of stress called?

26. The data obtained from a rectangular strain gauge rosette attached to a stressed steel member are

$$\varepsilon_0 = -220 \times 10^{-6},\ \varepsilon_{45^\circ} = 120 \times 10^{-6} \text{ and } \varepsilon_{90^\circ} = 220 \times 10^{-6}.$$

Given that the value of $E = 2 \times 10^6$ N/mm^2 and Poisson's Ratio $\mu = 0.3$, calculate the values of principal stresses acting at the point and their directions.

27. A 1 m × 1 m mild steel sheet of 1 mm thickness is stretched in its own plane by stresses $\sigma_x = 20$ MPa and $\sigma_y = 30$ MPa as shown in figure below. Point O is centre of the plate. OC and OD are two mutually perpendicular lines inclined at 45° each to x and y directions respectively before application of stress. Determine the change in angle (in degrees) between OC and OD after application of stresses. Take modulus of rigidity of plate material $G = 80$ GPa.



28. Two planes AB and AC make an angle of 50° at point A. Plane AB is subjected to tensile stress 3 kN/cm^2 and shear stress 3 kN/cm^2 from B towards A. Plane AC is subjected to a normal stress of unknown magnitude and a shear stress of magnitude 2 kN/cm^2 from C towards A. Determine

(i) normal stress on plane AC, and

(ii) principal stresses,

29. At a point in a stressed body, principal strains are ε_1 and ε_2. If E is the Young's modulus and ν is the Poisson's ratio, write down expressions for principal stresses in terms of ε_1, ε_2, E and ν.

30. In a loaded component principal stresses at a point are 130 MPa and 30 MPa. Using Mohr's stress circle, determine the state of stress at the point with respect to axes x' and y' which are inclined to σ_1 and σ_2 (principal stress) axes at 30° each in

anticlockwise direction. Also determine principal strains at the point. Take E = 200 GPa and G = 80 GPa.

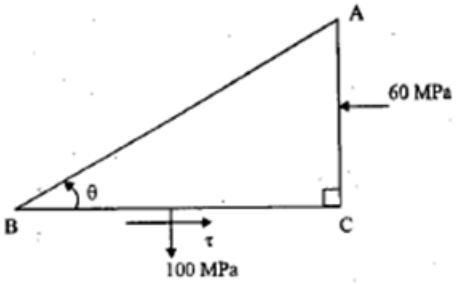
31. At a point in a loaded component the state of stress is given by σ_x = 270 MPa, σ_y = 130 MPa and $\tau_{xy} = \pm$ 40 MPa.

Determine

(i) the maximum and minimum principal stresses and the planes on which they act.

(ii) the maximum shearing stress in magnitude and direction.

32. Stresses at a point are shown in Fig. 1(b). What is the shear stress on plane AC, if τ on plane BC is 40 MPa? What is the magnitude of angle θ if AB is a principal plane? What is the maximum shear stress at the point?



चित्र 1(b)/Fig. 1(b)

33. Describe the Mohr's circle diagram as applied to two-dimensional stress, indicating principal dimensions and angles and what they represent.

34. The state of stress at a point in a structural component of elastic material are given as follows: A normal tensile stress of 160 MPa and a shearing stress of 120 MPa on one plane, a normal compressive stress of 110 MPa and the complementary shearing stress of 120 MPa on a second plane orthogonal to the first plane, and no stress on the third plane which is orthogonal to the above two planes.

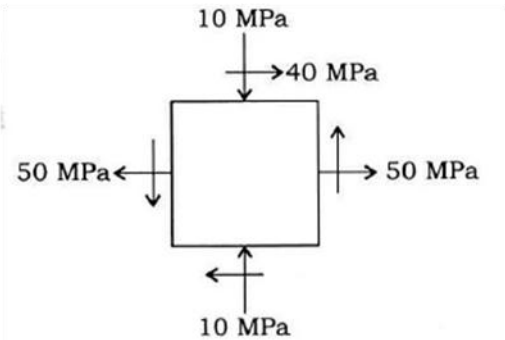
Determine:

(i) the principal stresses and the positions of the planes on which they act, and

(ii) the positions of planes on which there is no normal stress.

35. At a point in a material, the stresses on two mutually perpendicular planes are 60 N/mm² (tensile) and 40 N/mm² (tensile). The shear stress across these planes is 15 N/mm². Find the magnitude and direction of the resultant stress on a plane making an angle of 40° with the plane of the first stress. Also find the normal and tangential stresses on this plane.

36. A state of plane stress is shown in Fig. 1(b).



चित्र 1(b) / Fig. 1(b)

Determine the following:

(i) Principal stresses,

(ii) Principal planes,

(iii) Maximum shear stress.

37. The state of stress at a point in a loaded piece of material is given by

$$\sigma_x = 85 \text{ kPa}, \ \sigma_y = -40 \text{ kPa and } \tau_{xy} = \pm 50 \text{ kPa}.$$

Find the magnitudes of the principal stresses. Also find the magnitude of the maximum shear stress and the plane on which this acts.

38. Determine the maximum shear stress values for the given set of principal stresses. Draw the Mohr's stress circle for each case and show the maximum shear stress on it.

(A) $\sigma_1 = \sigma, \ \sigma_2 = \sigma/2, \ \sigma_3 = 0$

(B) $\sigma_1 = \sigma, \ \sigma_2 = -\sigma, \ \sigma_3 = 0$

(C) $\sigma_1 = \sigma, \ \sigma_2 = 0, \ \sigma_3 = 0$

(D) $\sigma_1 = \sigma_2 = \sigma_3 = \sigma$.

39. An element with stresses acting on it at a point is shown in Figure 2.

$\sigma_x = 80$ MPa

$\sigma_y = -100$ MPa

$\tau_{xy} = \pm 50$ MPa

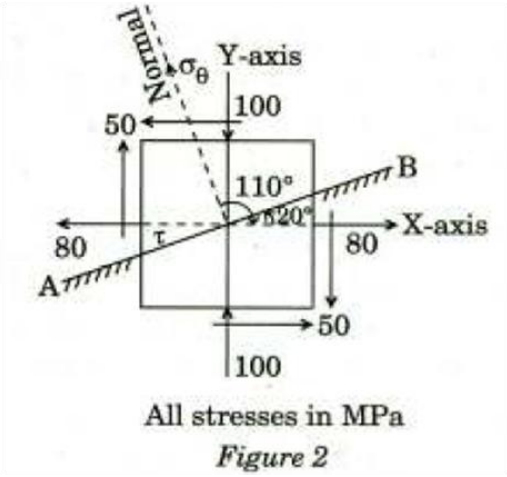


Figure 2

Determine:

(i) Normal and shear stresses acting on a plane *AB*, whose normal is at an angle of 110° w.r.t. *X*-axis.

(ii) Principal stresses and their location.

(iii) Maximum shear stress and its location.

40. The principal stresses at a point are given as per the following figure:

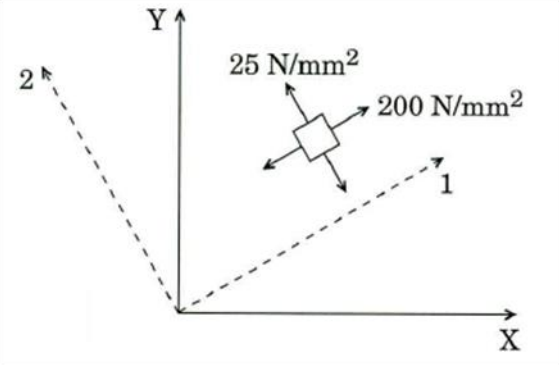


Figure Q 2(a)

Determine the state of stress w.r.t. *X* and *Y* axes. Also determine the maximum shear stress and direct stress on the plane of maximum shear.

41. Find the complementary shear stress for the element with stresses as shown in Figure 1(b) given below. The major principal stress is 120 MPa. What is the magnitude of maximum shear stress and minimum principal stress? Also, locate the principal planes and maximum shear stress planes.

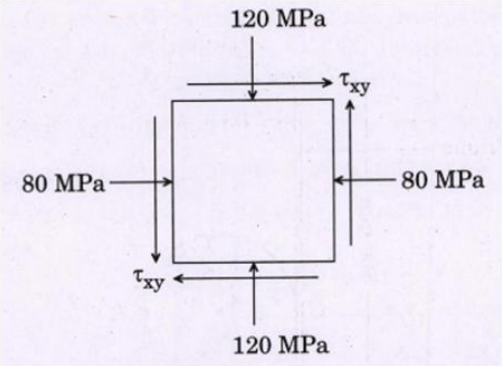


Figure 1(b)

42. At a point in a piece of elastic material, direct stresses of 90×10^6 N/m^2 tensile and 50×10^6 N/m^2 compressive are applied on mutually perpendicular planes. The planes are also subjected to shear stress. If the greater principal stress is limited to 100×10^6 N/m^2 tensile, determine—

(i) the value of the shear stress;
(ii) the other principal stress;
(iii) the normal stress on the plane of maximum shear;
(iv) the maximum shear stress.

Make a neat sketch showing clearly the positions of the principal planes and planes of maximum shear stress with respect to the planes of applied stresses.