Problems of Practices
Of
Mechanics of Solids
3- Theories of Elastic Failure

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1. Explain what you understand by theories of Failure. Compare any three Failure theories graphically for an element subjected to two mutually perpendicular direct stresses.

2. Explain the salient features of maximum distortion energy theory of elastic failure and discuss how it compares with maximum strain energy and maximum shear stress theories.

3. State and deduce the strain energy of distortion theory of elastic failure and compare it with maximum stress theory with respect to field of application and suitability for optimization.

4. For a two-dimensional stress state —
(i) Give the expressions specifying the acceptable stress domain according to the generally accepted theories of failure.
(ii) plot the domains of different theories in σ_1, σ_2 space.

5. A cantilever tube of length 120 mm is subjected to an axial tension P = 9.0 kN, a torsional moment T = 72.0 Nm, and a bending load F = 1.75 kN at the free end. The material is aluminum alloy with an yield strength of 276 MPa. Find the thickness of the tube limiting the outside diameter to 50 mm so as to ensure a factor of safety of 4.

6. Derive an expression for the distortion energy per unit volume for a body subjected to a uniform stress state, given by the principal stresses σ_1 and σ_2 with the third principal stress σ_3 being zero.

7. A round member is subjected to a direct tensile load of 20 kN and a shear load of 12 kN. The yield stress in tension is 25 kN/cm^2 and Poisson's ratio is 0.3. Determine the diameter of the member, using a factor of safety 2, according to:
(i) Maximum principal stress theory,
(ii) Maximum shear stress theory,
(iii) Maximum distortion energy theory.

8. State the theories of failure based on the following criteria:
(i) Maximum Stress,
(ii) Maximum Shear Stress,
(iii) Maximum Strain Energy,
(iv) Maximum distortion energy.
Compare the safe stress domains based on the above criteria illustrating your answer with σ_1, σ_2; safe domain plot for a two dimensional stress state. Compare the safe values based on each of the above theories for the specific case $\sigma_1 = -\sigma_2$.

9. A hollow circular steel shaft is subjected to a torque of 800 N-m and a bending moment of 1200 N-m. The internal diameter of the shaft is 60% of the external diameter. Determine the external diameter of the shaft according to:
(i) maximum principal stress theory,
(ii) maximum shear stress theory,
(iii) shear strain energy theory.
Take factor of safety as 2 and the yield strength of steel as 27 kN/cm^2.

10. A hollow shaft 30 mm inner diameter and 50 mm outer diameter is subjected to a twisting moment of 800 N-m and an axial compressive force of 40 kN. Determine the factor of safety according to theories of failure based on Normal stress theory, Maximum shear stress theory and Distortion energy theory. The tensile and compressive yield strength of material is 280 N/mm^2 and Poisson's ratio = 0.3.

11. A mild steel shaft of 50 mm diameter is subjected to a bending moment of 1.5 kN-m and torque T. If the yield point of steel in tension is 210 MPa, find the maximum value of torque without causing yielding of the shaft material according to
(i) Maximum Principal stress theory and
(ii) Maximum Shear stress theory.

12. What is the drawback of maximum principal stress theory?

13. A body is under action of two principal stresses of 40 N/ mm^2 and –70 N/mm^2, and the third principle stress being zero. If the elastic limit in simple tension as well as compression is 200 N/mm^2, find the factor of safety based on the elastic limit according to –
(i) maximum shear stress theory;
(ii) maximum strain energy theory;
(iii) maximum shear strain energy theory.
Take $\mu = 0.3$.

14. In a two-dimensional stress system, normal stresses of 30 MPa and 100 MPa act on two mutually perpendicular planes in conjunction with a shear stress of 40 MPa. The stress intensity, judged by the distortion energy, is excessive. As it was found impossible to reduce the applied stresses, the severity of the distortion ener-

gy condition was reduced by increasing the normal stress of 30 MPa to have higher value, S. Find the value of S at which the distortion energy minimum.

15. A hollow shaft whose internal diameter is half of the external diameter is subjected to maximum bending moment of 2 kN-m at a section and constant torque of 4 kN-m all along its length. If yield stress of the shaft material is 280 MPa and factor of safety is 3.0, what should be the minimum safe diameters of the shaft?

16. The stress state at a point in a body is plane with σ_1 = 60 N/mm^2 and σ_2 = –36 N/mm^2. If the allowable stress for the material in simple tension or compression is 100 N/mm^2 calculate the values of factor of safety with each of the following criteria for failure

(i) Max Stress Criteria,
(ii) Max Shear Stress Criteria,
(iii) Max Strain Criteria,
(iv) Max Distortion energy criteria.

17. A cast iron sample when tested in compression fails along approximately 45° plane from its axis while when tested in torsion also fails along a 45° (approx.) helical plane from its axis. Explain the reason for such failure and mention about the dominating stresses causing failure.

18. A hollow shaft of outside diameter 50 mm and inside diameter 20 mm is subjected to a torque of T N.m and a bending moment of 0.5T N.m. If the tensile yield stress of the shaft material is 250 N/mm^2, what is the maximum permissible value of T to avoid failure according to Tresca's failure theory? Take a factor of safety of 2.0 for a given application.

19. The load on a rod consists of an axial pull of 10 kN along with a transverse shear force of 5 kN. Determine the diameter of the rod by using the following theories of failure:

(i) Strain energy theory,
(ii) Shear strain energy theory.

Elastic limit in tension is 270 N/mm^2 and a factor of safety of 3 is to be used. Poisson's ratio = 0.3.

20. A cylindrical shaft, 80 mm in diameter, is subjected to a maximum bending moment of 2.5 kN-m and a twisting moment of 4.2 kN-m. Find the maximum principal stress developed in the shaft. If the yield stress of the shaft material is 380 MPa, determine the factor of safety of the shaft according to the maximum shearing stress theory of failure.

21. A solid circular shaft is subjected to a bending moment of 3 kN-m and a torque of 1 kN-m. The shaft is to be made in carbon steel for which σ_y = 480 MPa and τ_y = 265 MPa.

Calculate shaft diameter using:

(i) Maximum normal principal stress theory,
(ii) Maximum shear stress theory.

22. Determine the diameter of a circular shaft subjected to a bending moment M = 13 kN-m and a torque T = 30 kN-m according to maximum shear stress theory. Take σ_y = 700 MPa and use a Factor of safety of 2.6.

23. A state of stress at a point are given by

$$\sigma_x = 100 \text{ MPa}, \ \sigma_y = 80 \text{ MPa and } \tau_{xy} = \pm 50 \text{ MPa}.$$

Determine the principal stresses and the maximum shearing stress.

Take: E = 200 GPa, σ_y = 200 MPa and Poisson's ratio ν = 0.3.

Determine the factor of safety according to

(i) maximum principal stress theory,

(ii) maximum strain theory, and,

(iii) maximum shearing stress theory.

24. Compare the permissible diameter of a steel circular shaft, subjected to torsion, according to the following theories of failure. Assume Poisson's ratio to be 0.3:

(i) Maximum stress theory,

(ii) Maximum shear stress theory,

(iii) Maximum strain theory.

25. A shaft is subjected to a maximum torque of 10 kN-m and a maximum bending moment of 7.5 kN-m at a particular section. If the allowable equivalent stress in simple tension is 160 MN/m^2, find the diameter of the shaft according to (i) maximum shear stress theory, (ii) strain energy theory and (iii) shear strain energy theory. Take Poisson's ratio as 0.24.